4.4 調和振動子（結晶）

4.4.1 例2：古典系の調和振動子集団

cf. p.50

3番問題[1]

$$H(q,p) = \sum_i^N \left(\frac{1}{2m} p_i^2 + \frac{m\omega^2}{2} q_i^2 \right)$$

$$Z = \frac{1}{h^N} \int\cdots\int e^{-\sum_i \beta\left(\frac{1}{2m}p_i^2 + \frac{m\omega^2}{2}q_i^2\right)} dq_1 \cdots dq_N \, dp_1 \cdots dp_N$$

$$= \left(\frac{1}{h} \iint e^{-\beta\left(\frac{p^2}{2m} + \frac{m\omega^2}{2}q^2\right)} dp\, dq \right)^N$$

$$= \left(\frac{1}{h} \sqrt{\frac{2m\pi}{\beta}} \sqrt{\frac{2\pi}{\beta m\omega^2}} \right)^N$$

$$= \left(\frac{2\pi}{h} \frac{1}{\beta\omega} \right)^N$$

$$= \left(\frac{1}{\hbar\beta\omega} \right)^N$$

$$\therefore A = -k_B T \ln Z = N k_B T \ln \frac{\hbar\omega}{k_B T}$$

$$E = -T^2 \frac{\partial}{\partial T}\left(\frac{A}{T}\right) = N k_B T$$

$$S = -\frac{\partial A}{\partial T} = N k_B \left(1 - \ln \frac{\hbar\omega}{k_B T} \right)$$

$$\mu = \frac{\partial A}{\partial N} = k_B T \ln \frac{\hbar\omega}{k_B T}$$

$$C_V = \frac{\partial E}{\partial T} = N k_B$$