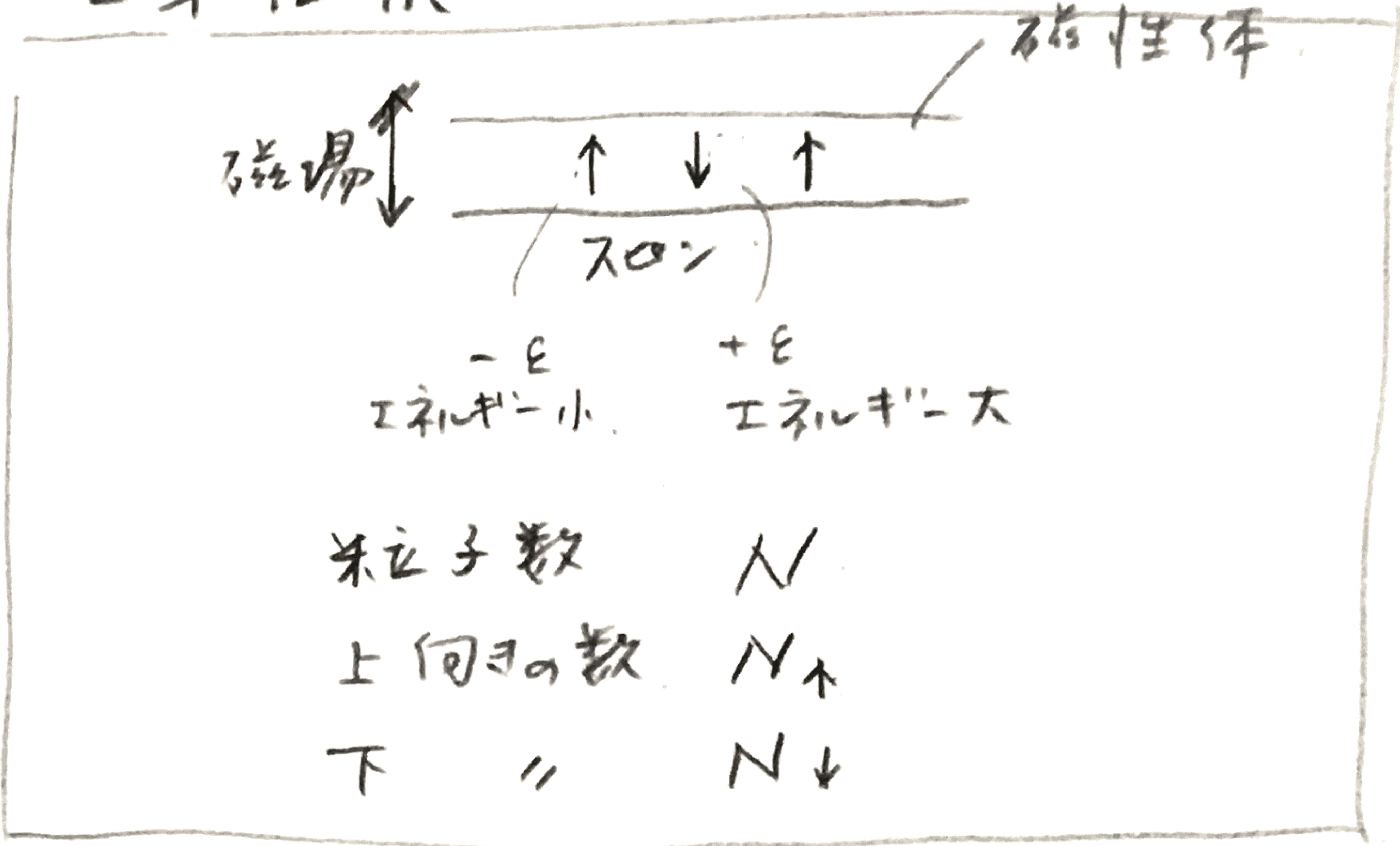


§3.4

2準位系



場合の数 $W(E, N) = \frac{N!}{N_{\uparrow}! N_{\downarrow}!}$

$E = \epsilon (N_{\downarrow} - N_{\uparrow})$
 $N = N_{\downarrow} + N_{\uparrow}$

① エンタルピー S

$S = k_B \ln W$

$= k_B (N \ln N - N_{\uparrow} \ln N_{\uparrow} - N_{\downarrow} \ln N_{\downarrow})$

$N_{\uparrow} = \frac{1}{2} (N - \frac{E}{\epsilon})$

$N_{\downarrow} = \frac{1}{2} (N + \frac{E}{\epsilon})$

$= k_B (N \ln N - \frac{1}{2} (N - \frac{E}{\epsilon}) \ln \frac{1}{2} (N - \frac{E}{\epsilon}) - \frac{1}{2} (N + \frac{E}{\epsilon}) \ln \frac{1}{2} (N + \frac{E}{\epsilon}))$

$$= k_B N \left(\cancel{\ln N} - \frac{1}{2} \left(1 - \frac{E}{\epsilon N}\right) \left(\cancel{\ln N} + \ln \frac{1}{2} \left(1 - \frac{E}{\epsilon N}\right)\right) - \frac{1}{2} \left(1 + \frac{E}{\epsilon N}\right) \left(\cancel{\ln N} + \ln \frac{1}{2} \left(1 + \frac{E}{\epsilon N}\right)\right) \right)$$

$$= -\frac{1}{2} k_B N \left(\left(1 - \frac{E}{\epsilon N}\right) \ln \frac{1}{2} \left(1 - \frac{E}{\epsilon N}\right) + \left(1 + \frac{E}{\epsilon N}\right) \ln \frac{1}{2} \left(1 + \frac{E}{\epsilon N}\right) \right)$$

② 温度 T

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E} \right)_{N, V}$$

$$= -\frac{1}{2} k_B N \left(-\frac{1}{\epsilon N} \ln \frac{1}{2} \left(1 - \frac{E}{\epsilon N}\right) + \left(1 - \frac{E}{\epsilon N}\right) \frac{2}{1 - \frac{E}{\epsilon N}} \cdot \left(-\frac{1}{2\epsilon N}\right) + \frac{1}{\epsilon N} \ln \frac{1}{2} \left(1 + \frac{E}{\epsilon N}\right) + \left(1 + \frac{E}{\epsilon N}\right) \frac{2}{1 + \frac{E}{\epsilon N}} \cdot \frac{1}{2\epsilon N} \right)$$

$$= -\frac{k_B}{2\epsilon} \left(\ln \frac{1}{2} \left(1 + \frac{E}{\epsilon N}\right) - \ln \frac{1}{2} \left(1 - \frac{E}{\epsilon N}\right) \right)$$

$$= -\frac{k_B}{2\epsilon} \left(\ln \left(1 + \frac{E}{\epsilon N}\right) - \ln \left(1 - \frac{E}{\epsilon N}\right) \right)$$

$$\therefore T = -\frac{2\epsilon}{k_B} \frac{1}{\ln \frac{1 + \frac{E}{\epsilon N}}{1 - \frac{E}{\epsilon N}}}$$

$$\ln \frac{1 + \frac{E}{\epsilon N}}{1 - \frac{E}{\epsilon N}} = -\frac{2\epsilon}{k_B T}$$

$$\therefore \frac{1 + \frac{E}{\epsilon N}}{1 - \frac{E}{\epsilon N}} = e^{-\frac{2\epsilon}{k_B T}}$$

$$\therefore \frac{E}{\epsilon N} = \frac{e^{-\frac{2\epsilon}{k_B T}} - 1}{e^{-\frac{2\epsilon}{k_B T}} + 1} = \frac{e^{-\frac{\epsilon}{k_B T}} - e^{\frac{\epsilon}{k_B T}}}{e^{-\frac{\epsilon}{k_B T}} + e^{\frac{\epsilon}{k_B T}}}$$

$$\therefore E = -\epsilon N \tanh \frac{\epsilon}{k_B T}$$

③ 比熱 $\frac{\partial E}{\partial T} = - \epsilon N \left(\operatorname{sech}^2 \frac{\epsilon}{k_B T} \right) \times \left(- \frac{\epsilon}{k_B} \frac{1}{T^2} \right)$
 $= N k_B \left(\frac{\epsilon}{k_B T} \right)^2 \operatorname{sech}^2 \frac{\epsilon}{k_B T}$

" $\epsilon \ll k_B T$ " 型 比熱
 Schottky

* $\ln 2 \approx 0.693$ $\ln 3 \approx 1.099$

Maxima 2

```
plot 2d ( -sinh(1/x) / cosh(1/x), [x, 0.4]);
plot 2d ( 1/x / x * sech(1/x)^2, [x, 0.4]);
```

7.11.2.1 =

```
taylor ( log(1+x), x, 0, 10 );
```