§4.3　例1：古典理想気体

$$H(q,p) = \sum_i^N \frac{1}{2m}\left(p_{ix}^2 + p_{iy}^2 + p_{iz}^2\right)$$

$$Z = \frac{1}{N!}\frac{1}{h^{3N}}\int\cdots\int dq_1 \cdots dq_{3N}\, dp_1 \cdots dp_{3N}\, e^{-\beta H}$$

$$= \frac{1}{N!}\frac{1}{h^{3N}} V^N \left(\int_{-\infty}^{\infty} e^{-\beta\frac{p^2}{2m}} dp\right)^{3N}$$

$$= \frac{1}{N!}\left(\frac{V}{h^3}\right)^N \left(\sqrt{\frac{2m}{\beta}\pi}\right)^{3N}$$

$$= \frac{1}{N!}\left(\frac{V(2\pi m k_B T)^{\frac{3}{2}}}{h^3}\right)^N$$

☆ $$\int_{-\infty}^{\infty} e^{-ax^2} dx = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \frac{1}{\sqrt{a}}\sqrt{\pi} = \sqrt{\frac{\pi}{a}}$$

☆ 一般に相互作用のない系では

$$Z = \sum_r e^{-\beta E_r}$$

$$= \sum_r e^{-\beta(e_1 + e_2 + \cdots e_N)_r}$$

$$= \left(\sum_r e^{-\beta e_{ir}}\right)^N$$

Zから求まる熱力学関数

$$A = -k_B T \ln Z$$
$$= k_B T \left(\ln N! - N \ln \frac{V(2\pi m k_B T)^{3/2}}{h^3} \right)$$
$$= k_B T \left(N \ln N - N - N \ln \frac{V(2\pi m k_B T)^{3/2}}{h^3} \right)$$

$$S = -\left(\frac{\partial A}{\partial T}\right)_{V,N} = -k_B N \left(\ln N - 1 - \ln \frac{V(2\pi m k_B T)^{3/2}}{h^3} - \frac{3}{2} \right)$$
$$= +k_B N \left(\frac{5}{2} - \ln N + \ln \frac{V(2\pi m k_B T)^{3/2}}{h^3} \right)$$

↓

サッカー テトロード (2.42)

$$P = -\left(\frac{\partial A}{\partial V}\right)_{T,N} = k_B T \cdot N \cdot \frac{1}{V} = \frac{N k_B T}{V}$$

↓

状態方程式.

$$\mu = \left(\frac{\partial A}{\partial N}\right)_{T,V} = k_B T \left(1 + \ln N - 1 - \ln \frac{V(2\pi m k_B T)^{3/2}}{h^3} \right)$$
$$= k_B T \left(\ln N - \ln \frac{V(2\pi m k_B T)^{3/2}}{h^3} \right)$$

$$E = A + TS$$
$$= \frac{3}{2} N k_B T$$