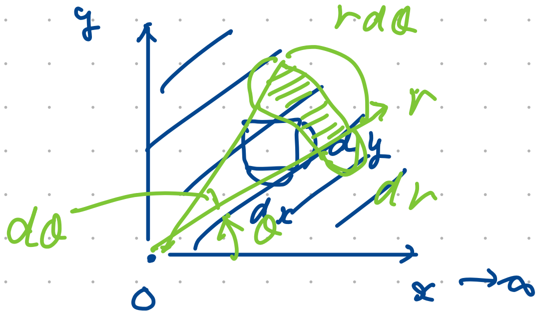
ガウス積分



$$I = \int_0^\infty e^{-x^2} dx$$

$$I = \int_0^\infty e^{-y^2} dy$$

$$I^2 = \int_0^\infty \int_0^\infty e^{-x^2} \times e^{-y^2} dx\, dy$$

$$= \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$$

極座標

$$dx\, dy \Rightarrow r\, dr\, d\theta$$

$$= \int_0^\infty \int_0^{\frac{\pi}{2}} \underbrace{e^{-r^2} \quad r}_{\theta\text{によらない}} d\theta\, dr$$

$$\int_0^{\frac{\pi}{2}} d\theta = \left[\theta\right]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$$= \frac{\pi}{2} \cdot \int_0^\infty r e^{-r^2} dr$$

$$= \frac{\pi}{2} \cdot \int_0^\infty -2re^{-r^2} dr \times \frac{1}{(-2)}$$

$$\frac{d}{dr} e^{-r^2} = \boxed{-2} r e^{-r^2}$$

$$= -\frac{\pi}{4} \left[e^{-r^2}\right]_0^\infty$$

$$= -\frac{\pi}{4} (0 - 1)$$

$$= \frac{\pi}{4}$$

$$\therefore I = \frac{\sqrt{\pi}}{2}$$